Work with your neighbor. (This will be graded for participation only.)

1. The following code to remove the last element in a linked list does not work for all linked lists:

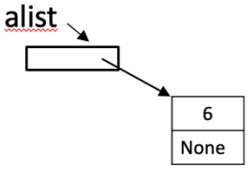
```
def remove_last(self):
    if self._head == None:    #the list is empty
        return None

    curr = self._head
    while curr._next != None:
        prev = curr
        curr = curr._next
    prev._next = None
    return curr
```

a) Give an example linked list that causes this code to crash with an error.

ANS:

This does not work for a list with one element:



b) Write a correct version of the code.

ANS:

```
def remove_last(self):
    if self._head == None:    #the list is empty
        return None

    curr = self._head
    if curr._next == None:
        self._head = None
        return curr

    while curr._next != None:
        prev = curr
```

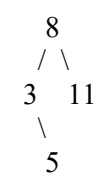
```
            curr = curr._next
        prev._next = None
        return curr
```

2. This function attempts to verify that a tree is a Binary Search Tree (BST). Unfortunately, it has a few bugs. (Note: an empty tree and a leaf are both BSTs.)

```
def is_BST(root):
    if root is None:
        return True
    if root._left is None and root._right is None:
        return True
    if root._left._val > root._val or
                root._right._val < root._val:
        return False
    return is_BST(root._left) and is_BST(root._right)
```

a) The function ends with an exception on many trees. Give an example of a tree that makes this function crash. Then fix that bug and write the fixed version below:

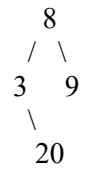
ANS:
It does not work for this tree, since it would look in `root._left._val` when the root is 3 and there is no left child:




```
def is_BST(root):
    if root is None:
        return True
    if root._left is None and root._right is None:
        return True
    if root._left is not None and root._left._value > root._value:
        return False
    if root._right is not None and root._right._value < root._value:
        return False
    return is_BST(root._left) and is_BST(root._right)
```

b) Does your fixed code work for all BSTs? If not, give an example of a tree for which your improved function will return an incorrect answer. (I.e., it returns `True` even if the tree is not a BST).

ANS:
No, it doesn't work for all trees. For example, it would return True for this tree:



c) Explain why your function does not work for your example tree.

ANS:
The code is only looking to see if a child follows the BST property as compared to its parent. So, for example, 20 is greater than 3, but not greater than the grandparent node 8.

d) How would you fix this problem? You don't need to write the code, just explain in a couple of sentences what you need to do for a correct solution.

ANS:
1)You could write functions to get the minimum and maximum values of a tree, then use those functions to make sure that the max of the left side of the tree is less than the value of the root node, and that the min of the right side is greater than the root node. You would have to check every node in the tree recursively.
2) You could generate the inorder traversal of the tree and then verify that it is in sorted order.

3. For each of the code snippets below, the assert on the last line sometimes throws an exception (an assertion error) and ends program execution. Which value(s) in this code should you print out in order to debug this? Why is the code sometimes failing?

    a) code snippet 1

```
data1 = …a Python list…
data2 = set(data1)
assert len(data1) == len(data2)
```

ANS:
The list may have duplicate elements, but the set of those elements will not have duplicates. That will make the lengths differ.

    b) code snippet 2

```
data1 = …a Python list…
data2 = sorted(data1)
assert data2[0] <= data2[1]
```

ANS:
The list may not have two or more elements. If it doesn't, the code will terminate with an index out-of-bounds error.

4. The calculation for BMI is weight in kilograms divided by the square of the height in meters. The function below computes the BMI given a weight and a height:

```
def calculate_bmi(weight, height):
    return weight / (height ** 2)
```

The following code produces the same BMI for the pairs of weights and heights in the `patients` list:

```
patients = [(70, 1.8), (80, 1.9), (150, 1.7)]
for patient in patients:
    weight, height = patients[0]
    bmi = calculate_bmi(height, weight)
    print("Patient's BMI is:", bmi)
```

Output:

```
Patient's BMI is: 0.00036734693877551024
Patient's BMI is: 0.00036734693877551024
Patient's BMI is: 0.00036734693877551024
```

What would you print out to find the bug? What is the bug?

ANS:
I would print out `weight` and `height`.

The assignment
`weight, height = patients[0]`
should be
`weight, height = patients`

5. The following function takes a list of strings `alist` and returns a list of all of the strings in `alist` where the first character matches the last character:

```
def first_matches_last(alist):
    match_list = []
    for elem in alist:
        if elem[0] == elem[-1]:
            match_list.append(elem)

    return match_list
```

a) Give an example list that causes this code to crash with an exception.

ANS: 1) The empty list `[]`. 2) A list that has an empty string as an element, `[ "eye", "", "hi"]`

b) Write a correct version of the code.

ANS:

```
def first_matches_last(alist):
    match_list = []
    for elem in alist:
        if len(elem) <= 1:
            continue
        if elem[0] == elem[-1]:
            match_list.append(elem)

    return match_list
```

6. The class `Word` defines a word object and implements `__eq__()` to return `True` if two `Word` objects are anagrams and `False` otherwise:

```
class Word:
    def __init__(self, word):
        self._word = word

    def __eq__(self, other):
        if len(self._word) == len(other._word):
            for letter in self._word.lower():
                if letter not in other._word.lower():
                    return False
        return True
```

The method works for the following examples:

```
w1 = Word("post")
w2 = Word("stop")
print(w1 == w2)

w1 = Word("keep")
w2 = Word("peep")
print(w1 == w2)
```

e) Does this code work for all words? If not, give an example of two words that give an incorrect answer for equality.

ANS: No. It does not work if the `self._word` attribute of the two `Word` objects are not the same length. (Returns `True`.)

f) Rewrite the method to fix any bugs you have found.

ANS:

```
def __eq__(self, other):
    if len(self._word) == len(other._word):
        for letter in self._word.lower():
            if letter not in other._word.lower():
                return False
        return True
    return False
```

Final exam review (various questions on complexity)

7. In lecture we covered a simple memory model and showed how Python lists are laid out in memory.
   (a) Based on that discussion, what is the fundamental reason that indexing into a Python list and appending to a Python list are both O(1)?

   ANS:
   They are O(1) because the elements (or to be precise, the references to the elements) are contiguous in memory. Since Python has a reference to the header of the list, to access a list element only requires doing some multiplication and addition. Likewise for append, there is space at the end to new elements.

   (b) Why is inserting into a Python list O(n)?

   ANS:
   Worst case, the insertion may be at position 0 and all of the elements of the list would need to be moved.

8. Write a function called `func(s)`, where `s` is a string, such that the complexity of `func` is O(n). Your function can have any purpose; just make it O(n).

ANS: There are many answers. Anything that involves potentially visiting every element of a list or string would be O(n), where n is the length of the sequence. Here is one example:

```
def func(s):
    digits = 0
    for c in s:
        if c in '0123456789':
            digits += 1
    return digits
```

9. What is the complexity of searching for a key in a hash table?
   ANS: O(1)